

EFFECTS OF BLOCKAGE AND EROSION ON THE FILTRATION OF SUSPENSIONS

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A model of filtration is considered which takes into account blockage (colmatation) and erosion (suffosion) of the pores using a generalized relationship between permeability and porosity.

It is known that the relaxation phenomena which occur during the filtration of heavy petroleum oils, polymer solutions, and other systems in porous media can be caused by various properties of the filtering liquid and of the porous medium itself. The establishment of the reasons for the dependence of the relaxation phenomena on these characteristics is a problem of basic importance in the theory of filtration, the solution of which would make it possible to carry out qualitative and quantitative analyses of the relaxation phenomena and to predict and control them based on practical requirements.

In the known hypothetical models of relaxation in filtration such a relationship has not been investigated in explicit form. Nevertheless, in a number of cases the models give completely acceptable results and identify new qualitative aspects of the filtration process [1-4].

Relaxation phenomena during the filtration of real liquids can be caused by various mechanisms. Up to now they have been studied with insufficient completeness, and estimates have not been made of their effects on the relaxation phenomena. Separate attempts to explain the mechanism of relaxation filtration have been made in [4-8]. As follows from the discussion in [6], one of the possible mechanisms is the "hardening" of the particles which are contained in the liquid being filtered and the "closing" of the pores by them. Other filtration mechanisms which lead to relaxation have been analyzed in [7]. It is shown that in [8] that the qualitative features of the filtration of suspensions and colloidal solutions consist of a reduction in the effective permeability of the porous medium as a result of the deposition of the particles of the disperse phase of the solution being filtered onto its surfaces. The reduction of the permeability of the porous medium is equivalent to the occurrence of an effective liquid flow which varies with time, which is reflected from the inlet layer of the porous medium. Thus, the effect of the closing of the pores by the particles of the disperse medium is one of the possible causes for the occurrence of the relaxation phenomena.

The purpose of the present work is to analyze the effect of blockage (colmatation) of the pores of the skeleton of the porous medium by impurity particles contained in the filtrate and their subsequent erosion (suffosion) on the features of the filtration flows. These impurities may be particles of tars, asphaltenes, paraffins, etc., contained in various petroleum oils, and solid particles of suspensions, the droplets in emulsions, etc. Such a problem has already been considered in [9], where, however, only a very generalized scheme of filtration was investigated. In schemes where choking (closing) occurs it is essential to take into account changes in porosity, and as a result of these, changes in permeability also. This was taken into account in [9] to a linear approximation. In the general case the dependence of the permeability on the porosity has a more complex form. Several formulas have been given in [10, 11] which express the filtration coefficient in terms of the porosity. Of these formulas the most widely used is the Carman-Kozeny formula

The process of closing the pores by the particles from the filtering liquid and freeing (suffosion) depend both on the geometry of the particles and pores and also on the dynamic

$$K(\varepsilon) = k \frac{\varepsilon^3}{(1-\varepsilon)^2}, \quad k = 8.2d^2/\mu \quad (1)$$

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conditions of filtration. Following [9], let us consider the process of filtration allowing for blockage and erosion for the case of a generalized relationship between the permeability and the porosity, modeling the porous medium by a continuum of traps for the contaminant particles. Each trap corresponds to one pore. If the pore is closed by a particle, it is assumed that it is captured by the trap.

The probability of capture W_0 by the trap (pore) of even one contaminant particle and of its being set free W_1 in unit time are assumed (following [9]) in the form:

$$W_0 = \omega_0, \quad W_1 = \omega_1 |\nabla p|, \quad (2)$$

where ω_0 and ω_1 are empirical constants; $|\nabla p|$ is the modulus of the pressure gradient. The selection of W_0 , W_1 in such a simplified form is explained by the attempt to obtain physical results. The first of the equations (2) means that the capture of the contaminant occurs even in the absence of a pressure drop, while the second equation means that the opening of the pore can occur only as a result of the removal of the obstructing particle under the influence of the applied pressure drop. However, in the general case the relationship (2) can be made more precise by taking into account various factors influencing the blockage and erosion.

A change in the number of free traps per unit time occurs as a result of taking into account the freeing of the occupied traps and the capture of contaminant by the free traps:

$$dN/dt = (N_0 - N) \omega_1 |\nabla p| - N \omega_0. \quad (3)$$

When allowance is made for a generalized dependence of the filtration coefficient on the porosity, Darcy's law assumes the form

$$v = K(\varepsilon) |\nabla p|, \quad (4)$$

where $K(\varepsilon)$ is given by Eq. (1).

The porosity of the medium ε is defined in terms of the concentration of pores N and their free volume β as $\varepsilon = \beta N$. Equation (3) can therefore be written in the form

$$d\varepsilon/dt = (\varepsilon_0 - \varepsilon) \omega_1 |\nabla p| - \varepsilon \omega_0. \quad (5)$$

where $\varepsilon_0 = \beta N$ is the porosity accessible for filtration when all the pore-traps are free.

Equations (4) and (5) form a system by means of which the change in the porosity ε with time is defined under unsteady-state filtration conditions. The filtration relationship for various conditions of loading of the system can be determined from Eq. (4). In principle it is not difficult to determine the relationship between v and ∇p for any arbitrary flow conditions. For simplicity, a typical case is considered below when a difference in either the pressure drop or the filtration rate is specified.

Difference in the Pressure Gradient. Suppose that a steady-state process is established in the system (in the porous medium) corresponding to an initial pressure drop $|\nabla p^0|$. At the initial moment of time $t = 0$ a shift is created in the pressure gradient from $|\nabla p^0|$ to $|\nabla p^1|$. The corresponding steady-state values of the porosity are denoted by ε^0 and ε^1 . In this case it is found from Eq. (5) that

$$\varepsilon = \varepsilon^1 + (\varepsilon^0 - \varepsilon^1) \exp(-t/\tau'), \quad \tau' = (\omega_1 |\nabla p^1| + \omega_0)^{-1}. \quad (6)$$

By substituting (6) into (4) and using the Garman-Kozeny formula, it is found that

$$v = k \frac{[\varepsilon^1 + (\varepsilon^0 - \varepsilon^1) \exp(-t/\tau')]^3}{[1 - \varepsilon^1 - (\varepsilon^0 - \varepsilon^1) \exp(-t/\tau')]^2} |\nabla p^1|. \quad (7)$$

In the general case, relationship (7) expresses Darcy's law with a filtration coefficient which varies with time. It can be represented in the form

$$v = \frac{k(\varepsilon^1)^3}{(1 - \varepsilon^1)^2} \times \frac{1 - a_1 \exp\left(-\frac{t}{\tau'}\right) + a_2 \exp\left(-\frac{2t}{\tau'}\right) - a_3 \exp\left(-\frac{3t}{\tau'}\right)}{1 + b_1 \exp\left(-\frac{t}{\tau'}\right) + b_2 \exp\left(-\frac{2t}{\tau'}\right)} |\nabla p^1|, \quad (8)$$

$$a_1 = 3\Omega, a_2 = 3\Omega^2, a_3 = \Omega^3, b_1 = 2\theta, b_2 = \theta^2,$$

$$\Omega = 1 - \varepsilon^0/\varepsilon', \theta = (\varepsilon' - \varepsilon^0)/(1 - \varepsilon').$$

According to Eq. (8), with a constant shift of the pressure gradient $|\nabla p'|$ the rate of filtration v relaxes to a new steady value corresponding to $\nabla p'$. The relaxation time τ' in Eq. (8) depends on $|\nabla p'|$, and consequently for different values of $\nabla p'$ the filtration rate v will relax with different characteristic relaxation times.

It is of interest to investigate the dynamics of v with respect to t under various conditions of loading of the system, and in particular when there is a step increase or decrease of ∇p . Suppose that a shift is made from ∇p^0 to $\nabla p'$ ($|\nabla p'| > |\nabla p^0|$). The relaxation time τ' is then determined by the value of $\nabla p'$ and its value will be smaller than the relaxation time corresponding to a transition process due to a shift ∇p from some value to ∇p^0 . This means that if the pressure gradient initially rises from ∇p^0 to $\nabla p'$ and then later falls again to ∇p^0 , the time to establish steady-state flow in the second case will be greater than in the first. This hysteresis-type relationship between v and ∇p is observed in the filtration of heavy petroleum oils and polymer solutions [12]. The considerable differences in the times of establishing pressures and depletions in reservoir models which are observed during filtration in these systems are also explained to some extent by this phenomenon.

The steady-state dependence of v on $|\nabla p|$ has the following form, as can be seen from Eq. (5),

$$v = \frac{k\varepsilon^3}{(1 - \varepsilon)^2} |\nabla p|. \quad (9)$$

Taking into account the steady-state value of the porosity ε corresponding to $|\nabla p|$, namely, $|\nabla p|: \varepsilon = \varepsilon_0 \omega_1 |\nabla p| / (\omega_1 |\nabla p| + \omega_0)$, it is found from Eq. (9) that

$$v = \frac{k(\varepsilon_0 \omega_1)^3 |\nabla p|^4}{(\omega_1 |\nabla p| + \omega_0) [\omega_1 (1 - \varepsilon_0) |\nabla p| + \omega_0]^2}. \quad (10)$$

The dependence of v on $|\nabla p|$ according to Eq. (10) is shown schematically in Fig. 1. At large pressure drops ∇p (or generally speaking, when $\omega_1 |\nabla p| \gg \omega_0$), the relationship has an asymptote

$$v = \frac{k\varepsilon_0^3}{(1 - \varepsilon_0)^2} \left(|\nabla p| - \frac{3 - \varepsilon_0}{1 - \varepsilon_0} \frac{\omega_0}{\omega_1} \right). \quad (11)$$

As can be seen from (11), the asymptote intersects the $|\nabla p|$ axis at $|\nabla p_0| = (3 - \varepsilon_0)\omega_0 / [(1 - \varepsilon_0)\omega_1]$, which can be interpreted as a limiting pressure gradient.

In the case of a linear dependence of the permeability (filtration coefficient) on the porosity, the asymptote intersects the $|\nabla p|$ axis at $|\nabla p_0| = \omega_0/\omega_1$. From the condition $(3 - \varepsilon_0)/(1 - \varepsilon_0) > 1$ it is obvious that the "limiting pressure gradient" is larger in the case of a nonlinear relationship for the permeability (filtration coefficient) than in the case of the analogous linear dependence. This fact can be regarded as the effect of the increase in the deviation from the classical Darcy's law in the case of a nonlinear relationship between the permeability (filtration coefficient) and the porosity.

Difference in the Flow Rate. Now assume that the rate of filtration shifts from v^0 to v' , and the pressure gradient relaxes to a new constant value.

By substituting (4) into (5) and use of the Carman-Kozeny equation (1) the following equation is arrived at:

$$\frac{d\varepsilon}{dt} = \left[(\varepsilon_0 - \varepsilon) \frac{(1 - \varepsilon)^2}{\varepsilon^3} \gamma - \varepsilon \right] \omega_0, \quad \gamma = \frac{\omega_1 v'}{\omega_0 k}. \quad (12)$$

By integrating (12), it is found that

$$\int_{\varepsilon_0}^{\varepsilon} \frac{\xi^3 d\xi}{\xi^4 + \gamma \xi^3 - \gamma(2 + \varepsilon_0) \xi^2 + \gamma(2\varepsilon_0 + 1) \xi - \gamma \varepsilon_0} = -\omega_0 t, \quad (13)$$

where ε^0 is the steady-state value of the porosity corresponding to the rate of filtration v^0

Equation (13) makes it possible to find the change in ε at various values of t . The particular value of ε is introduced as the upper limit of the integral (13). Evaluating the integral (13) in explicit form depending on the nature of the solution of the equation (with respect to ξ)

$$\xi^4 + \gamma \xi^3 - \gamma(2 + \varepsilon_0) \xi^2 + \gamma(2\varepsilon_0 + 1) \xi - \gamma \varepsilon_0 = 0, \quad (14)$$

produces the equation for the determination of ε .

1. Suppose that the roots of (14) are real and different. Then carrying out the integration in (13) with the initial condition of $\varepsilon = \varepsilon^0$ at $t = 0$, the following equation is found with respect to ε :

$$\prod_{n=1}^4 |\varepsilon - \alpha_n|^{\beta_n} = \exp(-\omega_0 t) \prod_{n=1}^4 |\varepsilon^0 - \alpha_n|^{\beta_n}, \quad (15)$$

where β_n are the coefficients for expanding the function under the integral as a vulgar fraction, i.e.,

$$\frac{\xi^3}{\xi^4 + \gamma \xi^3 - \gamma(2 + \varepsilon_0) \xi^2 + \gamma(2\varepsilon_0 + 1) \xi - \gamma \varepsilon_0} = \sum_{n=1}^4 \frac{\beta_n}{\xi - \alpha_n}.$$

2. Suppose that Eq. (14) has two real roots α_1, α_2 and two complex conjugate roots $\alpha_3 = \alpha_3' + \alpha_3''i, \alpha_4 = \alpha_3' - \alpha_3''i, i = \sqrt{-1}$. Then by carrying out the integration in (13), the following equation is obtained for determining ε :

$$\ln \left(\left| \frac{\varepsilon - \alpha_1}{\varepsilon^0 - \alpha_1} \right|^{\beta_1} \left| \frac{\varepsilon - \alpha_2}{\varepsilon^0 - \alpha_2} \right|^{\beta_2} \left| \frac{\varepsilon^2 - \delta_1 \varepsilon + \delta_2}{(\varepsilon^0)^2 - \delta_1 \varepsilon^0 + \delta_2} \right|^{\beta_3/2} \right) + \frac{\beta_4 + \beta_3 \alpha_3'}{\alpha_3''} \left(\arctg \frac{\varepsilon - \alpha_3'}{\alpha_3''} - \arctg \frac{\varepsilon^0 - \alpha_3'}{\alpha_3''} \right) = -\omega_0 t, \quad (16)$$

$$\delta_1 = 2\alpha_3', \quad \delta_2 = (\alpha_3')^2 + (\alpha_3'')^2.$$

The case of two pairs of complex conjugate roots of Eq. (14) does not occur.

Equations (15) and (16) are solved numerically. The results of the calculations according to Eq. (16) with shifts in the filtration rate corresponding to changes in the parameter γ from 0.1 to 0.6, 0.8, or 1.0 are shown in Fig. 2. As can be seen from this figure, a steady-state value of the porosity of $\varepsilon^0 = 0.28$ corresponds to $\gamma = 0.1$. As γ shifts to 0.6, 0.8, and 1.0, the porosity relaxes to the corresponding steady-state values of 0.345, 0.353, and 0.359.

The rate of change of $\varepsilon(t)$ is determined by the value of the relaxation time. As can be seen from Eqs. (15) and (16), it is possible to select ω_0^{-1} as the characteristic time which governs the rate of change of ε with respect to time. The curves in Fig. 2 show clearly that the change in ε with time occurs according to a relaxation relationship.

Calculations of the change in porosity can also be carried out with Eq. (13). It is interesting to note that the steady-state value of the porosity corresponding to the filtration rate v' is the root of Eq. (14). Consequently, the integral in (13) will be an improper integral. When $t \rightarrow \infty$ it is obvious that $\varepsilon \rightarrow \varepsilon'$, and the value of the integral tends to $-\infty$. When $t \rightarrow \infty$ it is found analogously in Eqs. (15) and (16) that $\varepsilon \rightarrow \alpha_n$ for any value of n . The asymptotics of Eqs. (15) and (16) for $t \rightarrow \infty$ characterize the stage of the emergence into the steady state of filtration corresponding to the specified shift of the velocity v' .

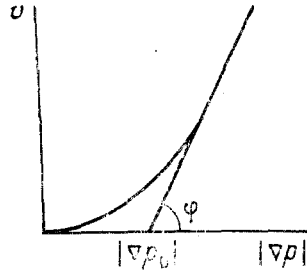


Fig. 1. Steady-state dependence of the flow value v on the pressure gradient $|\nabla p|$; $|\nabla p_0|$ is the point of intersection of the asymptote with the axis; $|\nabla p|$ is the limiting pressure gradient; $\tan \phi = k\varepsilon_0^3/(1 - \varepsilon_0)^2$.

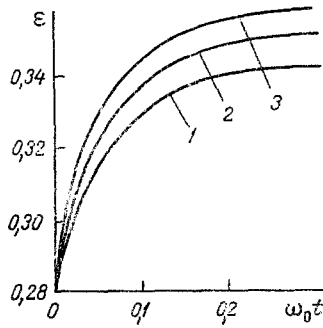


Fig. 2

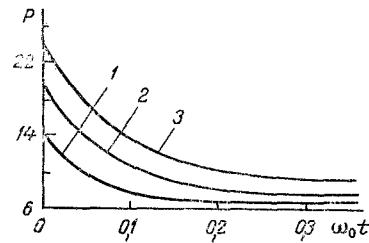


Fig. 3

Fig. 2. Relaxation of the porosity ε as v changes from v^0 to v^1 for various values of $\gamma = \omega_1 v^1 / (\omega_0 k)$: 1) $\gamma = 0.6$; 2) $\gamma = 0.8$; 3) $\gamma = 1.0$.

Fig. 3. Dependence of $P = \omega_1 |\nabla p| / \omega_0$ on $\omega_0 t$ for changes of γ from 0.1 to 0.6 (curve 1), 0.8 (curve 2), or 1.0 (curve 3).

The calculation of the change in the pressure gradient is carried out from the formula

$$|\nabla p| = \frac{(1 - \varepsilon)^2}{k\varepsilon^3} v', \quad (17)$$

where use is made of $\varepsilon = \varepsilon(t)$ which is found as the solution of Eq. (15) or (16). In dimensionless units, Eq. (17) assumes the form

$$P = \frac{(1 - \varepsilon)^2}{\varepsilon^3} \gamma, \quad P = \frac{\omega_1 |\nabla p|}{\omega_0}. \quad (18)$$

Let us consider the change in the pressure gradient for a step-wise change of γ from one value to another. By using the calculations of $\varepsilon = \varepsilon(t)$ given above for changes of γ from 0.1 to 0.6, 0.8, and 1.0, the curves for the changes in the parameter P calculated by Eq. (18) are given in Fig. 3 as a function of $\omega_0 t$. As can be seen from this last figure the dimensionless pressure gradient decreases monotonically to some constant value corresponding to the steady state. At large values of the shift in γ the initial and local values of P are correspondingly increased. When γ is reduced from one constant value to another, the pressure gradient is calculated analogously. As can be seen from Eqs. (15) and (16) the characteristic time of the process transition for a change of the filtration rate is defined as ω_0^{-1} . In contrast to the case when there is a shift in the pressure gradient, in this case the relaxation time is a constant quantity, and does not depend on the value of the pressure gradient.

Thus, taking into account the nonlinearity of the dependence of the permeability (filtration coefficient) on the porosity leads to a number of special features in the way in which the nature of the filtration process varies when blockage and erosion are taken into account.

It should be noted that generalization of the arguments given above to other relationships between the permeability (filtration coefficient) and the porosity does not present any special difficulty. Thus, the problem which has been considered can be easily extended to the formulas of E. A. Zamarin and I. I. Zauerbrei and other empirical relationships linking the permeability and the porosity, information on which is given in [10, 11]. Any difficulties which arise in doing this will be of a purely computational nature.

Further improvements of the model which has been proposed can be made in several directions. It seems that it would be very interesting to establish a more detailed structure of the probability of capture W_0 by a trap of even a single particle of the impurity and of its liberation W_1 per unit time. This would make it possible to carry out a more detailed investigation of the physical phenomena occurring in the process of filtration taking into account blockage, erosion, and other phenomena which lead to relaxation-type changes in the filtration characteristics under dynamic conditions. In particular, the latter could include complex rheological properties of the skeleton of the porous medium and of the filtering liquid, interactions of the liquid with the walls of the pores, and complex structures of the porous media (crack formation, nonhomogeneity, anomalously low permeability, etc.).

NOTATION

d , effective diameter; $K(\epsilon)$, filtration coefficient; k , coefficient in Eq. (1); N , number of free traps; N_0 , number of traps, equal to the number of pores per unit volume of the medium; p , pressure; v , filtration rate; W_0 , probability of capturing (W_1 is the probability of setting free) even one particle by a trap; t , time; ϵ , porosity; ϵ^0 , ϵ^1 , porosities corresponding to the steady states specified by $|v_p^0|$ (v^0) and $|v_p^1|$ (v^1); ϵ_0 , porosity accessible for filtration when all the pores are open; μ , viscosity of the filtrate; ω_0 , ω_1 , empirical constants in Eq. (2); τ' , relaxation time of the porosity, defined in Eq. (6).

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